Transverse Spin Structure/What is Orbital Angular Momentum?

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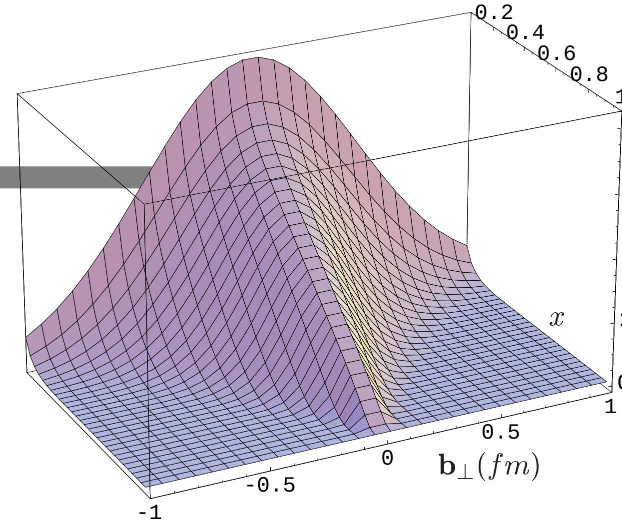
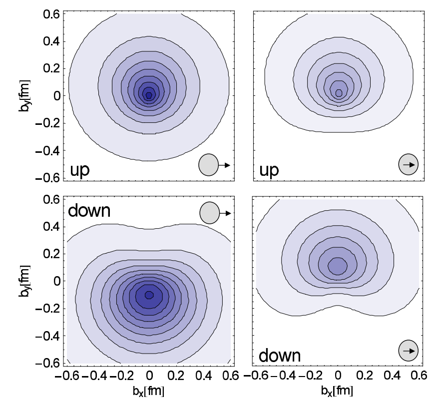
Outline

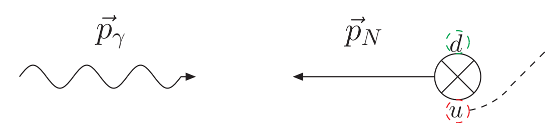


- Probabilistic interpretation of GPDs as Fourier trafos of impact parameter dependent PDFs
  - $H(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow q(x,\mathbf{b}_\perp)$
  - $\tilde{H}(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow \Delta q(x,\mathbf{b}_\perp)$
  - $E(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow$ $\perp$ distortion of PDFs when the target is $\perp$ polarized



- Chromodynamik lensing and $\perp$ SSAs

$$\left.\begin{array}{c}\text{transverse distortion of PDFs}\\ +\text{ final state interactions}\end{array}\right\} \Rightarrow \perp \text{SSA in } \gamma N \longrightarrow \pi + X$$



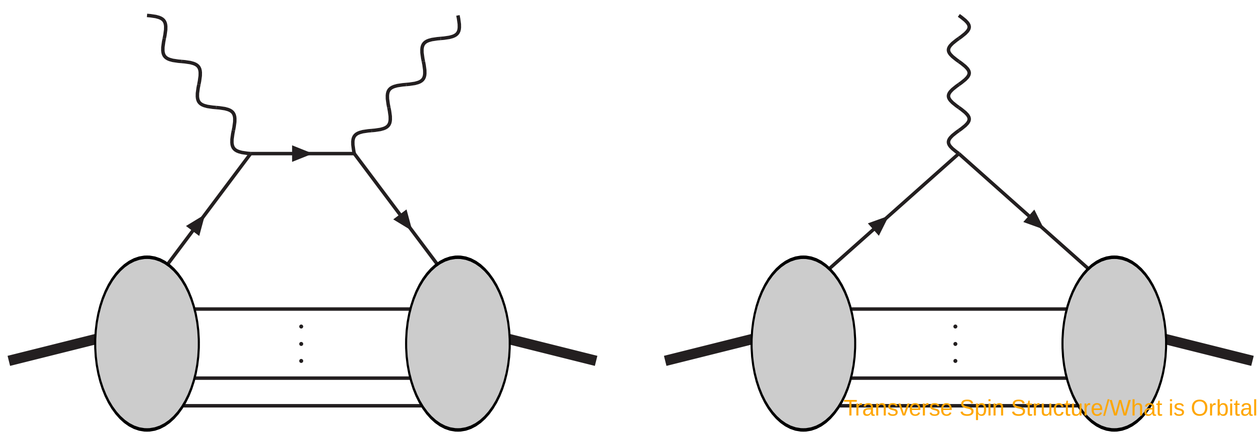
- Orbital angular momentum for an electron in QED
- Summary

Generalized Parton Distributions (GPDs)

- GPDs: **decomposition of form factors** at a given value of t, w.r.t. the average momentum fraction $x = \frac{1}{2}(x_i + x_f)$ of the active quark

$$
\begin{aligned}
\int dx H_q(x,\xi,t) &= F_1^q(t) \qquad \int dx \tilde{H}_q(x,\xi,t) = G_A^q(t) \\
\int dx E_q(x,\xi,t) &= F_2^q(t) \qquad \int dx \tilde{E}_q(x,\xi,t) = G_P^q(t),
\end{aligned}
$$

  - x_i and x_f are the momentum fractions of the quark before and after the momentum transfer
  - $2\xi = x_f - x_i$
- GPDs can be probed in deeply virtual Compton scattering (DVCS)



Generalized Parton Distributions (GPDs)

- formal definition (unpol. quarks):

$$\int \frac{dx^-}{2\pi} e^{ix^- \bar{p}^+ x} \left\langle p' \left| \bar{q}\left(-\frac{x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right) \right| p \right\rangle = H(x,\xi,\Delta^2)\bar{u}(p')\gamma^+ u(p)$$

$$+E(x,\xi,\Delta^2)\bar{u}(p')\frac{i\sigma^{+\nu}\Delta_\nu}{2M}u(p)$$

- in the limit of vanishing t and ξ, the nucleon non-helicity-flip GPDs must reduce to the ordinary PDFs:

$$H_q(x,0,0) = q(x) \qquad \tilde{H}_q(x,0,0) = \Delta q(x).$$

- DVCS amplitude

$$\mathcal{A}(\xi,t) \sim \int_{-1}^{1} \frac{dx}{x-\xi+i\varepsilon} GPD(x,\xi,t)$$

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi}\bar{q}\left(\frac{-x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,\xi,t)$	?

Form Factors vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,0,t)$	$q(x,\mathbf{b}_\perp)$

$q(x,\mathbf{b}_\perp)$ = impact parameter dependent PDF

Impact parameter dependent PDFs

- define $\perp$ localized state [D.Soper,PRD**15**, 1141 (1977)]

$$\left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \lambda\right\rangle \equiv \mathcal{N} \int d^2\mathbf{p}_\perp \left|p^+, \mathbf{p}_\perp, \lambda\right\rangle$$

  Note: $\perp$ boosts in IMF form Galilean subgroup $\Rightarrow$ this state has $\mathbf{R}_\perp \equiv \frac{1}{P^+} \int dx^- d^2\mathbf{x}_\perp \, \mathbf{x}_\perp T^{++}(x) = \sum_i x_i \mathbf{r}_{i,\perp} = \mathbf{0}_\perp$
  (cf.: working in CM frame in nonrel. physics)

- define impact parameter dependent PDF

$$q(x, \mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi} \left\langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp \right| \bar{q}(-\frac{x^-}{2}, \mathbf{b}_\perp) \gamma^+ q(\frac{x^-}{2}, \mathbf{b}_\perp) \left|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp\right\rangle e^{ixp^+x^-}$$

$$\hookrightarrow \boxed{\begin{aligned} q(x, \mathbf{b}_\perp) &= \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} e^{i\boldsymbol{\Delta}_\perp \cdot \mathbf{b}_\perp} H(x, 0, -\boldsymbol{\Delta}_\perp^2), \\ \Delta q(x, \mathbf{b}_\perp) &= \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} e^{i\boldsymbol{\Delta}_\perp \cdot \mathbf{b}_\perp} \tilde{H}(x, 0, -\boldsymbol{\Delta}_\perp^2), \end{aligned}}$$

Impact parameter dependent PDFs

- No relativistic corrections (Galilean subgroup!)

$\hookrightarrow$ corrolary: interpretation of 2d-FT of $F_1(Q^2)$ as charge density in transverse plane also free from relativistic corrections

- $q(x, \mathbf{b}_\perp)$ has probabilistic interpretation as number density ($\Delta q(x, \mathbf{b}_\perp)$ as difference of number densities)
- Reference point for IPDs is transverse center of (longitudinal) momentum $\mathbf{R}_\perp \equiv \sum_i x_i \mathbf{r}_{i,\perp}$

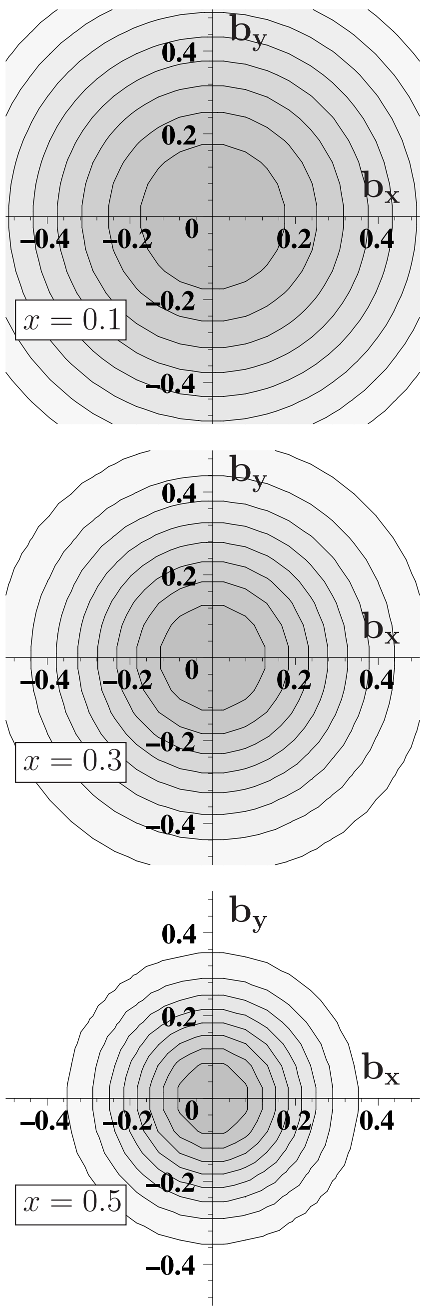
$\hookrightarrow$ for $x \rightarrow 1$, active quark 'becomes' COM, and $q(x, \mathbf{b}_\perp)$ must become very narrow (δ-function like)

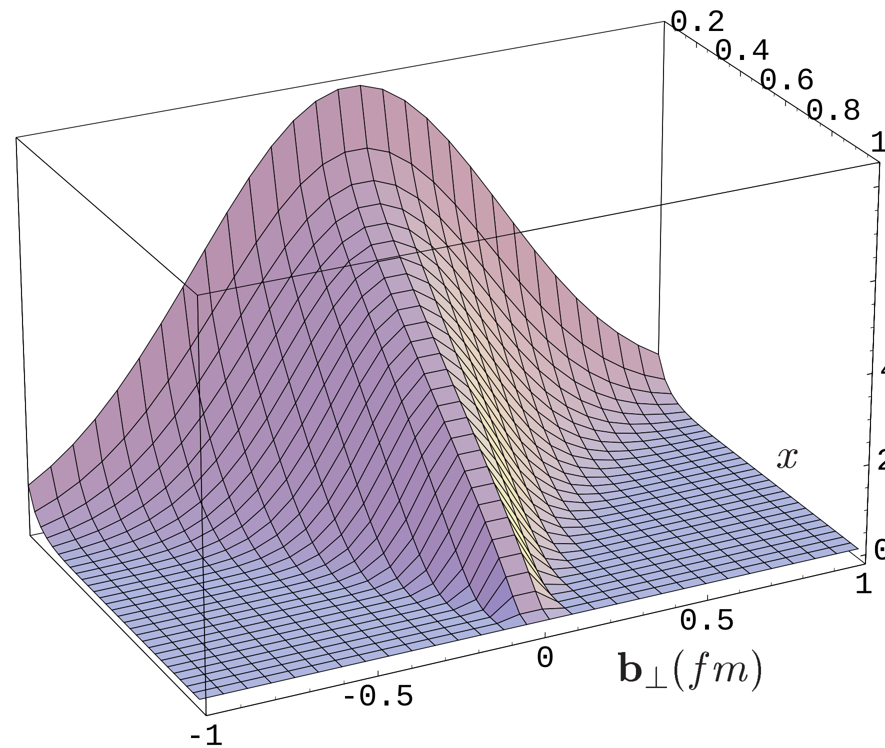
$\hookrightarrow$ $H(x, 0, -\mathbf{\Delta}_\perp^2)$ must become $\mathbf{\Delta}_\perp$ indep. as $x \rightarrow 1$ (MB, 2000)

$\hookrightarrow$ consistent with lattice results for first few moments

- Note that this does not necessarily imply that 'hadron size' goes to zero as $x \rightarrow 1$, as separation $\mathbf{r}_\perp$ between active quark and COM of spectators is related to impact parameter $\mathbf{b}_\perp$ via $\mathbf{r}_\perp = \frac{1}{1-x}\mathbf{b}_\perp$.

$q(x, \mathbf{b}_\perp)$ for unpol. p





$x =$ momentum fraction of the quark

$\vec{b} = \perp$ position of the quark

Transversely Deformed Distributions and $E(x, 0, -\mathbf{\Delta}_\perp^2)$

M.B., Int.J.Mod.Phys.A**18**, 173 (2003)

- So far: only unpolarized (or long. pol.) nucleon! In general ($\xi = 0$):

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P{+}\Delta,\uparrow \left| \bar{q}(0)\,\gamma^+ q(x^-) \right| P,\uparrow\rangle = H(x,0,-\mathbf{\Delta}_\perp^2)$$

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P{+}\Delta,\uparrow \left| \bar{q}(0)\,\gamma^+ q(x^-) \right| P,\downarrow\rangle = -\frac{\Delta_x - i\Delta_y}{2M} E(x,0,-\mathbf{\Delta}_\perp^2).$$

- Consider nucleon polarized in x direction (in IMF) $|X\rangle \equiv |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \uparrow\rangle + |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \downarrow\rangle$.

$\hookrightarrow$ unpolarized quark distribution for this state:

$$q(x,\mathbf{b}_\perp) = \mathcal{H}(x,\mathbf{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} E(x, 0, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$$

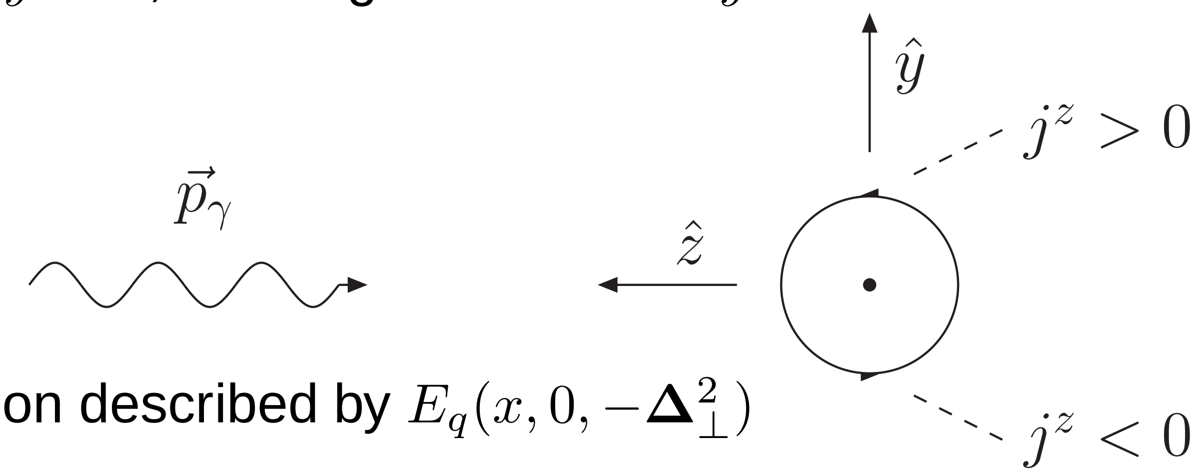
- Physics: $j^+ = j^0 + j^3$, and left-right asymmetry from j^3 ! [X.Ji, PRL **91**, 062001 (2003)]

Intuitive connection with $\vec{J}_q$

- DIS probes quark momentum density in the infinite momentum frame (IMF). Quark density in IMF corresponds to $j^+ = j^0 + j^3$ component in rest frame ($\vec{p}_{\gamma^*}$ in $-\hat{z}$ direction)

$\hookrightarrow$ j^+ larger than j^0 when quark current towards the γ^*; suppressed when away from γ^*

$\hookrightarrow$ For quarks with positive orbital angular momentum in $\hat{x}$-direction, j^z is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side



- Details of $\perp$ deformation described by $E_q(x, 0, -\mathbf{\Delta}_\perp^2)$

$\hookrightarrow$ not surprising that $E_q(x, 0, -\mathbf{\Delta}_\perp^2)$ enters Ji relation!

$$\langle J_q^i \rangle = S^i \int dx \left[H_q(x,0,0) + E_q(x,0,0) \right] x.$$

Transversely Deformed PDFs and $E(x, 0, -\mathbf{\Delta}_\perp^2)$

- $q(x, \mathbf{b}_\perp)$ in $\perp$ polarized nucleon is deformed compared to longitudinally polarized nucleons !
- mean $\perp$ deformation of flavor q ($\perp$ flavor dipole moment)

$$d_y^q \equiv \int dx \int d^2\mathbf{b}_\perp q_X(x, \mathbf{b}_\perp) b_y = \frac{1}{2M} \int dx E_q(x, 0, 0) = \frac{\kappa_q^p}{2M}$$

with $\kappa_{u/d}^p \equiv F_2^{u/d}(0) = \mathcal{O}(1-2) \quad \Rightarrow \quad d_y^q = \mathcal{O}(0.2 fm)$

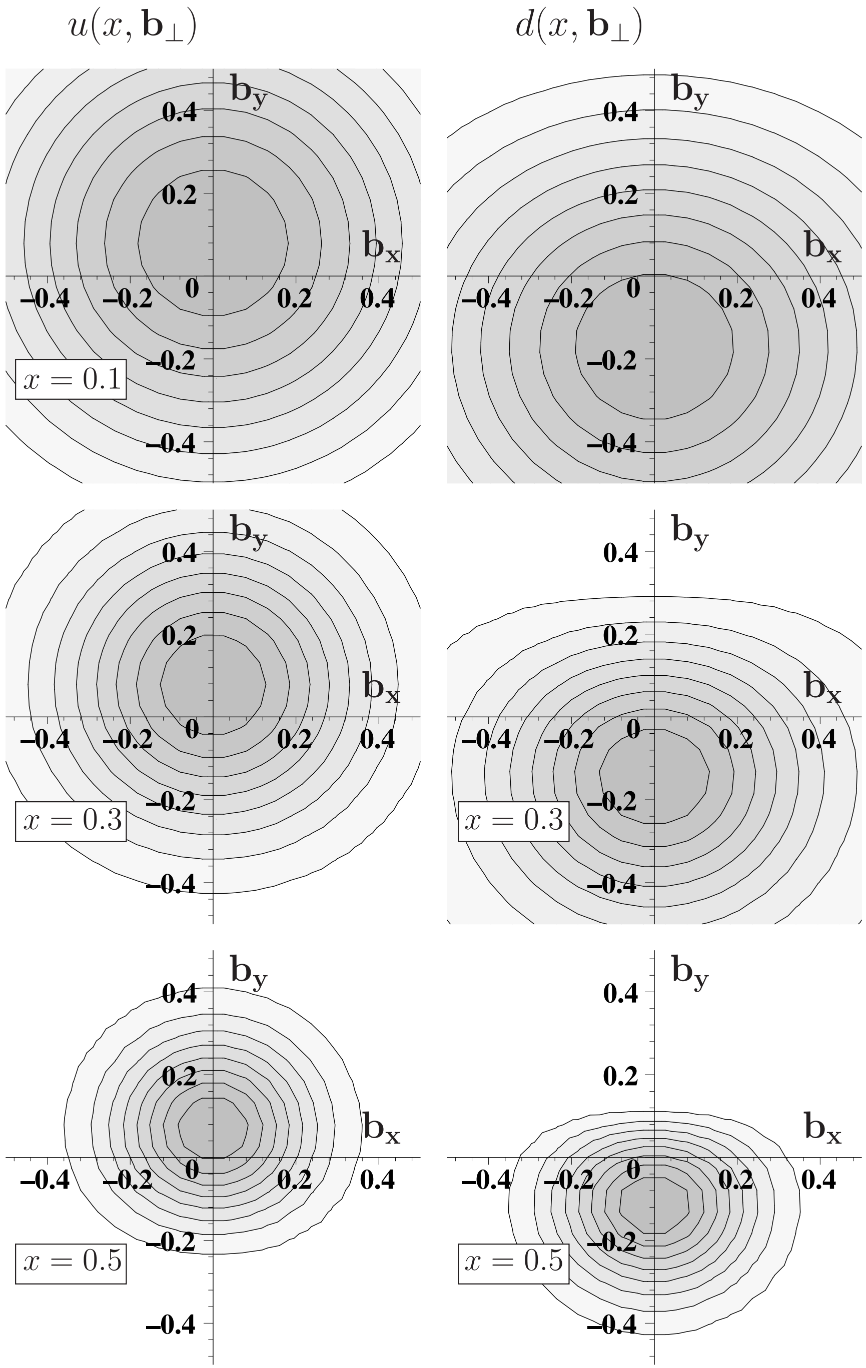
- simple model: for simplicity, make ansatz where $E_q \propto H_q$

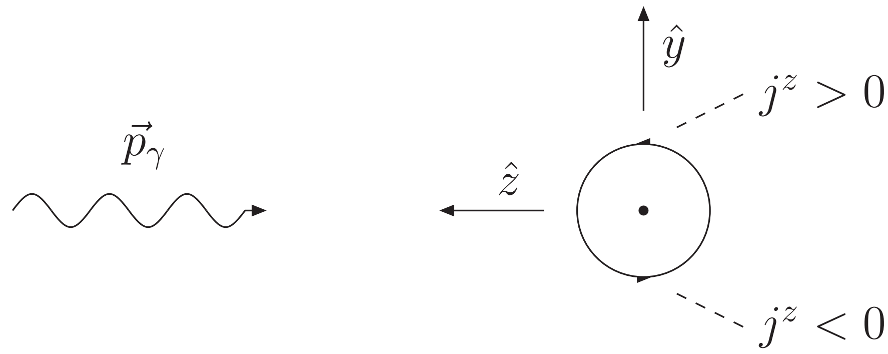
$$\begin{aligned} E_u(x, 0, -\mathbf{\Delta}_\perp^2) &= \frac{\kappa_u^p}{2} H_u(x, 0, -\mathbf{\Delta}_\perp^2) \\ E_d(x, 0, -\mathbf{\Delta}_\perp^2) &= \kappa_d^p H_d(x, 0, -\mathbf{\Delta}_\perp^2) \end{aligned}$$

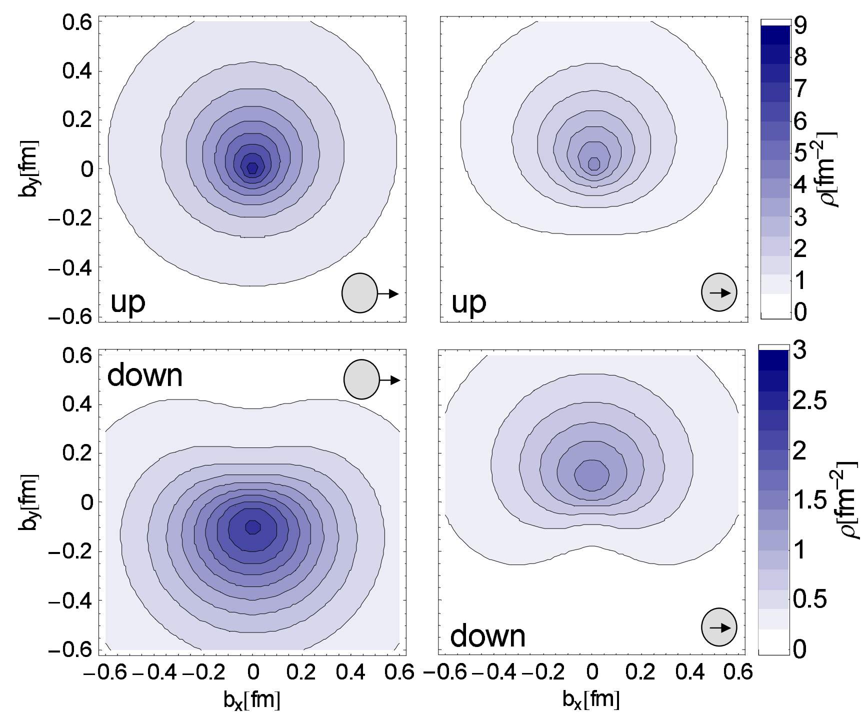
with $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673 \qquad \kappa_d^p = 2\kappa_n + \kappa_p = -2.033.$

- Model too simple but illustrates that anticipated deformation is very significant since κ_u and κ_d known to be large!

p polarized in $+\hat{x}$ direction



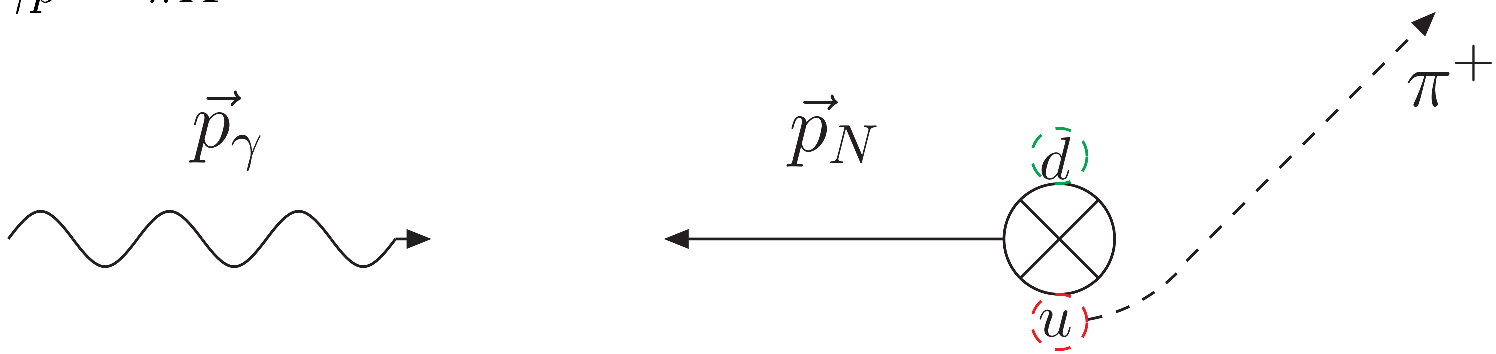




lattice results (Hägler et al.)

GPD $\longleftrightarrow$ SSA (Sivers)

- example: $\gamma p \to \pi X$



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign "determined" by κ_u & κ_d
-

attractive FSI deflects active quark towards the center of momentum

$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction

$\hookrightarrow$ correlation between sign of κ_q^p and sign of SSA: $f_{1T}^{\perp q} \sim -\kappa_q^p$

- $f_{1T}^{\perp q} \sim -\kappa_q^p$ confirmed by HERMES data (also consistent with COMPASS deuteron data $f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0$)

Quark-Gluon Correlations (Introduction)

- (longitudinally) polarized polarized DIS at leading twist $\longrightarrow$ ‘polarized quark distribution’ $g_1^q(x) = q^\uparrow(x) + \bar{q}^\uparrow(x) - q_\downarrow(x) - \bar{q}_\downarrow(x)$
- $\frac{1}{Q^2}$-corrections to X-section involve ‘higher-twist’ distribution functions, such as $g_2(x)$
- $g_2(x)$ involves quark-gluon correlations and does not have a parton interpretation as difference between number densities

Quark-Gluon Correlations (Introduction)

- (chirally even) higher-twist PDF $g_2(x) = g_T(x) - g_1(x)$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}(0)\gamma^\mu\gamma_5\psi(\lambda n)|_{Q^2}|PS\rangle$$
$$= 2\left[g_1(x,Q^2)p^\mu(S\cdot n) + g_T(x,Q^2)S_\perp^\mu + M^2 g_3(x,Q^2)n^\mu(S\cdot n)\right]$$

- 'usually', contribution from g_2 to polarized DIS X-section kinematically suppressed by $\frac{1}{\nu}$ compared to contribution from g_1

$$\sigma_{TT} \propto g_1 - \frac{2Mx}{\nu} g_2$$

- for $\perp$ polarized target, g_1 and g_2 contribute equally to σ_{LT}

$$\sigma_{LT} \propto g_T \equiv g_1 + g_2$$

$\hookrightarrow$ 'clean' separation between higher order corrections to leading twist (g_1) and higher twist effects (g_2)

- what can one learn from g_2?

Quark-Gluon Correlations (QCD analysis)

- (chirally even) higher-twist PDF $g_2(x) = g_T(x) - g_1(x)$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}(0)\gamma^\mu\gamma_5\psi(\lambda n)|_{Q^2}|PS\rangle$$
$$= 2\left[g_1(x,Q^2)p^\mu(S\cdot n) + g_T(x,Q^2)S_\perp^\mu + M^2 g_3(x,Q^2)n^\mu(S\cdot n)\right]$$

- $g_2(x) = g_2^{WW}(x) + \bar{g}_2(x)$, with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$
- $\bar{g}_2(x)$ involves quark-gluon correlations, e.g.

$$\int dx x^2 \bar{g}_2(x) = \frac{1}{3} d_2 = \frac{1}{6MP^{+2}S^x} \left\langle P,S\left|\bar{q}(0)gG^{+y}(0)\gamma^+ q(0)\right|P,S\right\rangle$$

- $\sqrt{2}G^{+y} \equiv G^{0y} + G^{zy} = -E^y + B^x$
- matrix elements of $\bar{q}B^x\gamma^+ q$ and $\bar{q}E^y\gamma^+ q$ are sometimes called color-electric and magnetic polarizabilities
  $2M^2\vec{S}\chi_E = \left\langle P,S\left|\vec{j}_a \times \vec{E}_a\right|P,S\right\rangle$ & $2M^2\vec{S}\chi_B = \left\langle P,S\left|j_a^0\vec{B}_a\right|P,S\right\rangle$
  with $d_2 = \frac{1}{4}(\chi_E + 2\chi_M)$ — but these names are misleading!

Quark-Gluon Correlations (Interpretation)

- $\bar{g}_2(x)$ involves quark-gluon correlations, e.g.

$$\int dx x^2 \bar{g}_2(x) = \frac{1}{3} d_2 = \frac{1}{6MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0) g G^{+y}(0) \gamma^+ q(0) \right| P,S \right\rangle$$

- QED: $\bar{q}(0)eF^{+y}(0)\gamma^+ q(0)$ correlator between quark density $\bar{q}\gamma^+ q$ and ($\hat{y}$-component of the) Lorentz-force

$$F^y = e\left[\vec{E} + \vec{v} \times \vec{B}\right]^y = e\left(E^y - B^x\right) = -e\left(F^{0y} + F^{zy}\right) = -e\sqrt{2}F^{+y}.$$

for charged paricle moving with $\vec{v} = (0,0,-1)$ in the $-\hat{z}$ direction

$\hookrightarrow$ matrix element of $\bar{q}(0)eF^{+y}(0)\gamma^+ q(0)$ yields γ^+ density (density relevant for DIS in Bj limit!) weighted with the Lorentz force that a charged particle with $\vec{v} = (0,0,-1)$ would experience at that point

$\hookrightarrow$ d_2 a measure for the **color Lorentz force** acting on the struck quark in SIDIS in the instant **after being hit by the virtual photon**

$$\langle F^y(0) \rangle = -M^2 d_2 \qquad \text{(rest frame; } S^x = 1\text{)}$$

Quark-Gluon Correlations (Interpretation)

- Interpretation of d_2 with the transverse FSI force in DIS also consistent with $\langle k_\perp^y \rangle \equiv \int_0^1 dx \int \mathrm{d}^2 k_\perp\, k_\perp^2 f_{1T}^\perp(x, k_\perp^2)$ in SIDIS (Qiu, Sterman)

$$\langle k_\perp^y \rangle = -\frac{1}{2p^+} \left\langle P, S \left| \bar{q}(0) \int_0^\infty dx^- g G^{+y}(x^-) \gamma^+ q(0) \right| P, S \right\rangle$$

  semi-classical interpretation: average $k_\perp$ in SIDIS obtained by correlating the quark density with the transverse impulse acquired from (color) Lorentz force acting on struck quark along its trajectory to (light-cone) infinity

- matrix element defining d_2 same as the integrand (for $x^- = 0$) in the QS-integral:
  - $\langle k_\perp^y \rangle = \int_0^\infty dt F^y(t)$ (use $\mathrm{d}x^- = \sqrt{2}\mathrm{d}t$)

  $\hookrightarrow$ first integration point $\longrightarrow F^y(0)$

  $\hookrightarrow$ (transverse) force at the begin of the trajectory, i.e. at the moment after absorbing the virtual photon

Quark-Gluon Correlations (Interpretation)

- x^2-moment of twist-4 polarized PDF $g_3(x)$
  $\int dx x^2 g_3(x) \rightsquigarrow \left\langle P,S \left| \bar{q}(0) g\tilde{G}^{\mu\nu}(0)\gamma_\nu q(0) \right| P,S \right\rangle \sim f_2$

$\hookrightarrow$ different linear combination $f_2 = \chi_E - \chi_B$ of χ_E and χ_M

$\hookrightarrow$ combine with data for $g_2 \Rightarrow$ disentangle electric and magnetic force

$\hookrightarrow$ combining JLab(E99-117)/SLAC(E155x) data this yields

- proton:
  $\chi_E = -0.082 \pm 0.016 \pm 0.071 \quad \chi_B = 0.056 \pm 0.008 \pm 0.036$
- neutron:
  $\chi_E = 0.031 \pm 0.005 \pm 0.028 \quad \chi_B = 0.036 \pm 0.034 \pm 0.017$

but future higher-Q^2 data for d_2 may still change these results ...

Quark-Gluon Correlations (Estimates)

- What should one expect (magnitude)?
  - if all spectators were to pull in the same direction, force should be on the order of the QCD string tension $\sigma \approx (0.45GeV)^2 \approx 0.2GeV^2$
  - however, expect significant cancellation for FSI force, from spectators 'pulling' in different directions

  $\hookrightarrow$ expect FSI force to be suppressed compared to string tension by about one order of magnitude (more?)

  $\hookrightarrow$ $|d_2| = \frac{|\langle F^y(0)\rangle|}{M^2} \sim 0.02$
- What should one expect (sign)?
  - $\kappa_q^p \longrightarrow$ signs of deformation (u/d quarks in $\pm\hat{y}$ direction for proton polarized in $+\hat{x}$ direction $\longrightarrow$ expect force in $\mp\hat{y}$

  $\hookrightarrow$ d_2 positive/negative for u/d quarks in proton
  - d_2 negative/positive for u/d quarks in neutron
  - large N_C: $d_2^{u/p} = -d_2^{d/p}$
  - consistent with $f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0$

Quark-Gluon Correlations (data/lattice)

- lattice (Göckeler et al.): $d_2^u \approx 0.010$ and $d_2^d \approx -0.0056$ (with large errors)

$\hookrightarrow$ using $M^2 \approx 5\frac{\text{GeV}}{fm}$ this implies

$$\langle F_u^y(0)\rangle \approx -50\frac{\text{MeV}}{fm} \qquad\qquad \langle F_d^y(0)\rangle \approx 28\frac{\text{MeV}}{fm}$$

- signs consistent with impact parameter picture
- SLAC data ($5GeV^2$): $d_2^p = 0.007 \pm 0.004$, $d_2^n = 0.004 \pm 0.010$
- combined with SIDIS data for $\langle k^y\rangle$, should tell us about 'effective range' of FSI $R_{eff} \equiv \frac{\langle k^y\rangle}{F^y(0)}$
  Anselmino et al.: $\langle k^y\rangle \sim \pm 100\,\text{MeV}$
- x^2-moment of chirally odd twist-3 PDF $e(x) \longrightarrow$ **transverse force on transversly polarized quark in unpolarized target** ($\leftrightarrow$ Boer-Mulders $h_1^\perp$)

Summary

- GPDs $\stackrel{FT}{\longleftrightarrow}$ IPDs (impact parameter dependent PDFs)
- $E(x, 0, -\boldsymbol{\Delta}_\perp^2) \longrightarrow \perp$ deformation of PDFs for $\perp$ polarized target
- $\hookrightarrow$ $\kappa^{q/p} \Rightarrow$ sign of deformation
- $\hookrightarrow$ attractive FSI $\Rightarrow f_{1T}^{\perp u} < 0$ & $f_{1T}^{\perp d} > 0$
- Interpretation of $M^2 d_2 \equiv 3M^2 \int dx x^2 \bar{g}_2(x)$ as $\perp$ **force on active quark in DIS in the instant after being struck by the virtual photon**

$$\langle F^y(0) \rangle = -M^2 d_2 \qquad \text{(rest frame; } S^x = 1\text{)}$$

- In combination with measurements of f_2
  - color-electric/magnetic force $\frac{M^2}{4}\chi_E$ and $\frac{M^2}{2}\chi_M$
- $\kappa^{q/p} \Rightarrow \perp$ deformation $\Rightarrow d_2^{u/p} > 0$ & $d_2^{d/p} < 0$ (attractive FSI)
- combine measurement of d_2 with that of $f_{1T}^\perp \Rightarrow$ **range of FSI**
- x^2-moment of chirally odd twist-3 PDF $e(x) \longrightarrow$ **transverse force on transversly polarized quark in unpolarized target** ($\longleftrightarrow$ Boer-Mulders $h_1^\perp$)

Summary

- distribution of $\perp$ polarized quarks in unpol. target described by chirally odd GPD $\bar{E}_T^q = 2\bar{H}_T^q + E_T^q$

$\hookrightarrow$ origin: correlation between orbital motion and spin of the quarks

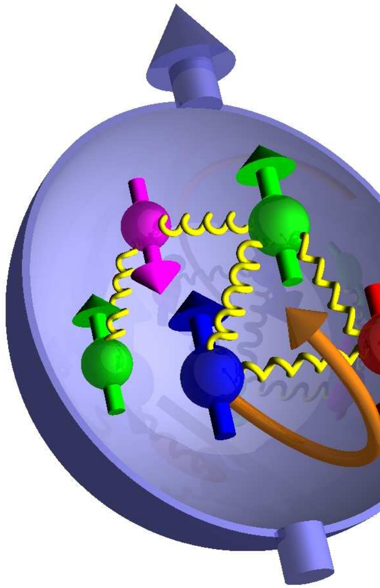
$\hookrightarrow$ attractive FSI $\Rightarrow$ measurement of $h_1^\perp$ (DY,SIDIS) provides information on $\bar{E}_T^q$ and hence on spin-orbit correlations

- expect:

$$h_1^{\perp,q} < 0 \qquad\qquad |h_1^{\perp,q}| > |f_{1T}^q|$$

- x^2-moment of chirally odd twist-3 PDF $e(x) \longrightarrow$ **transverse force on transversly polarized quark in unpolarized target** ($\longrightarrow$ Boer-Mulders)

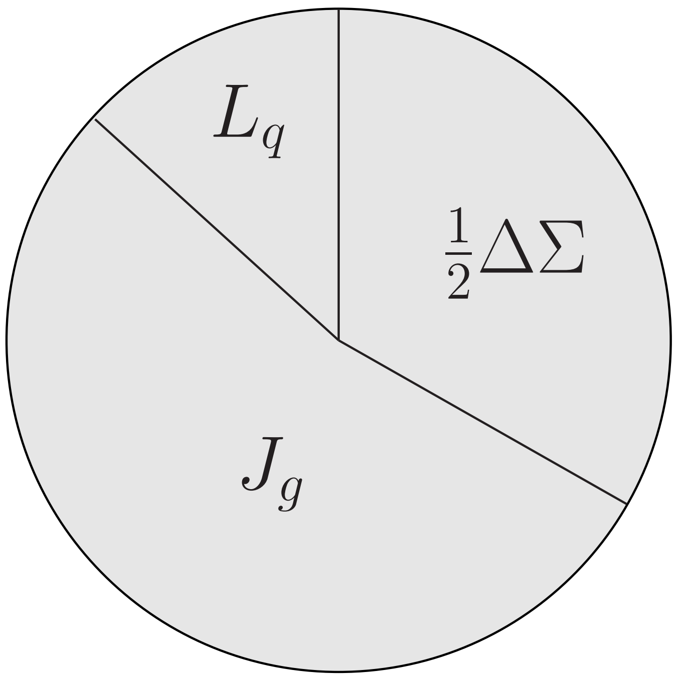
What is Orbital Angular Momentum?



- Ji decomposition
- Jaffe decomposition
- recent lattice results (Ji decomposition)
- model/QED illustrations for Ji v. Jaffe

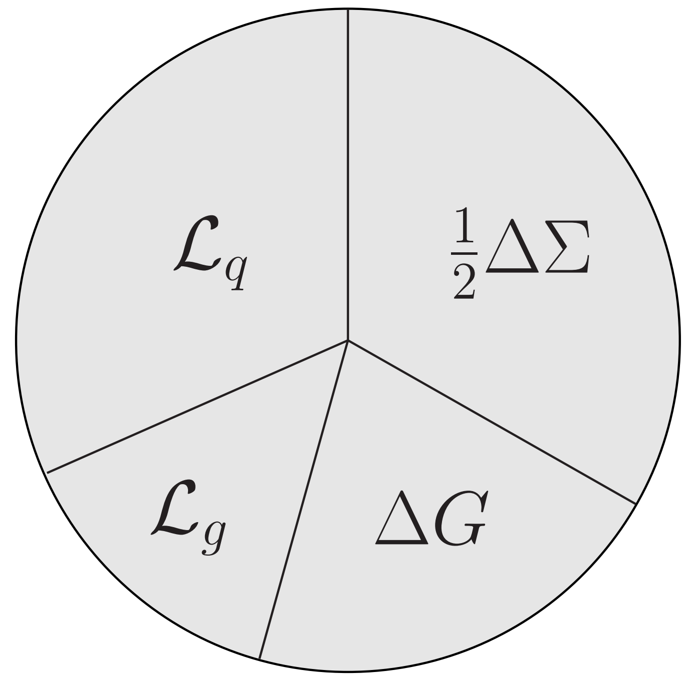
The nucleon spin pizza(s)

Ji



'pizza tre stagioni'

Jaffe & Manohar



'pizza quattro stagioni'

- only $\frac{1}{2}\Delta\Sigma \equiv \frac{1}{2}\sum_q \Delta q$ common to both decompositions!

Angular Momentum Operator

- angular momentum tensor $M^{\mu\nu\rho} = x^\mu T^{\nu\rho} - x^\nu T^{\mu\rho}$
- $\partial_\rho M^{\mu\nu\rho} = 0$

$\hookrightarrow$ $\tilde{J}^i = \frac{1}{2}\varepsilon^{ijk}\int d^3r M^{jk0}$ conserved

$$\frac{d}{dt}\tilde{J}^i = \frac{1}{2}\varepsilon^{ijk}\int d^3x \partial_0 M^{jk0} = \frac{1}{2}\varepsilon^{ijk}\int d^3x \partial_l M^{jkl} = 0$$

- $M^{\mu\nu\rho}$ contains time derivatives (since $T^{\mu\nu}$ does)
  - use eq. of motion to get rid of these (as in T^{0i})
  - integrate total derivatives appearing in T^{0i} by parts
  - yields terms where derivative acts on x^i which then 'disappears'

  $\hookrightarrow$ J^i usally contains both
    - 'Extrinsic' terms, which have the structure '$\vec{x}\times$ Operator', and can be identified with 'OAM'
    - 'Intrinsic' terms, where the factor $\vec{x}\times$ does not appear, and can be identified with 'spin'

Angular Momentum in QCD (Ji)

- following this general procedure, one finds in QCD

$$\vec{J} = \int d^3x \left[\psi^\dagger \vec{\Sigma}\psi + \psi^\dagger \vec{x} \times \left(i\vec{\partial} - g\vec{A}\right)\psi + \vec{x} \times \left(\vec{E} \times \vec{B}\right)\right]$$

  with $\Sigma^i = \frac{i}{2}\varepsilon^{ijk}\gamma^j\gamma^k$

- Ji does <u>not</u> integrate gluon term by parts, <u>nor</u> identify gluon spin/OAM separately
- Ji-decomposition valid for all three components of $\vec{J}$, but usually only applied to $\hat{z}$ component, where the quark spin term has a partonic interpretation

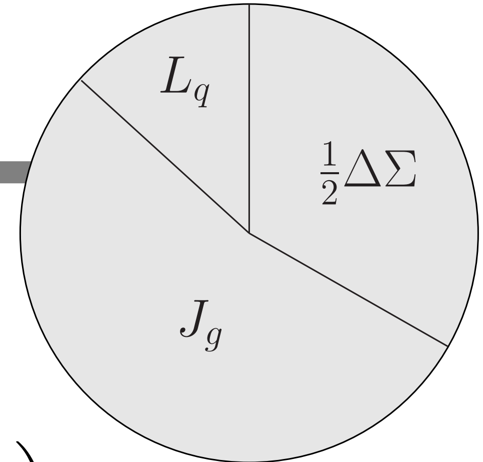
(+) all three terms manifestly gauge invariant

(+) DVCS can be used to probe $\vec{J}_q = \vec{S}_q + \vec{L}_q$

(-) quark OAM contains interactions

(-) only quark spin has partonic interpretation as a single particle density

Ji-decomposition



- Ji (1997)

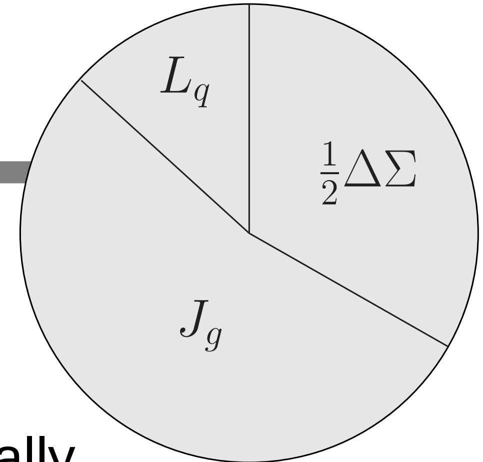
$$\frac{1}{2} = \sum_q J_q + J_g = \sum_q \left(\frac{1}{2}\Delta q + L_q \right) + J_g$$

with ($P^\mu = (M,0,0,1)$, $S^\mu = (0,0,0,1)$)

$$\begin{aligned} \frac{1}{2}\Delta q &= \frac{1}{2}\int d^3x \, \langle P,S| \, q^\dagger(\vec{x})\Sigma^3 q(\vec{x}) \, |P,S\rangle \qquad \Sigma^3 = i\gamma^1\gamma^2 \\ L_q &= \int d^3x \, \langle P,S| \, q^\dagger(\vec{x}) \left(\vec{x} \times i\vec{D}\right)^3 q(\vec{x}) \, |P,S\rangle \\ J_g &= \int d^3x \, \langle P,S| \left[\vec{x} \times \left(\vec{E} \times \vec{B}\right)\right]^3 |P,S\rangle \end{aligned}$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

Ji-decomposition



- $\vec{J} = \sum_q \frac{1}{2} q^\dagger \vec{\Sigma} q + q^\dagger \left(\vec{r} \times i\vec{D} \right) q + \vec{r} \times \left(\vec{E} \times \vec{B} \right)$ applies to each vector component of nucleon angular momentum, but Ji-decomposition usually applied only to $\hat{z}$ component where at least quark spin has parton interpretation as difference between number densities
- Δq from polarized DIS
- $J_q = \frac{1}{2}\Delta q + L_q$ from exp/lattice (GPDs)
- L_q in principle independently defined as matrix elements of $q^\dagger \left(\vec{r} \times i\vec{D} \right) q$, but in practice easier by subtraction $L_q = J_q - \frac{1}{2}\Delta q$
- J_g in principle accessible through gluon GPDs, but in practice easier by subtraction $J_g = \frac{1}{2} - J_q$
- further decomposition of J_g into intrinsic (spin) and extrinsic (OAM) that is local and manifestly gauge invariant has not been found

L_q for proton from Ji-relation (lattice)

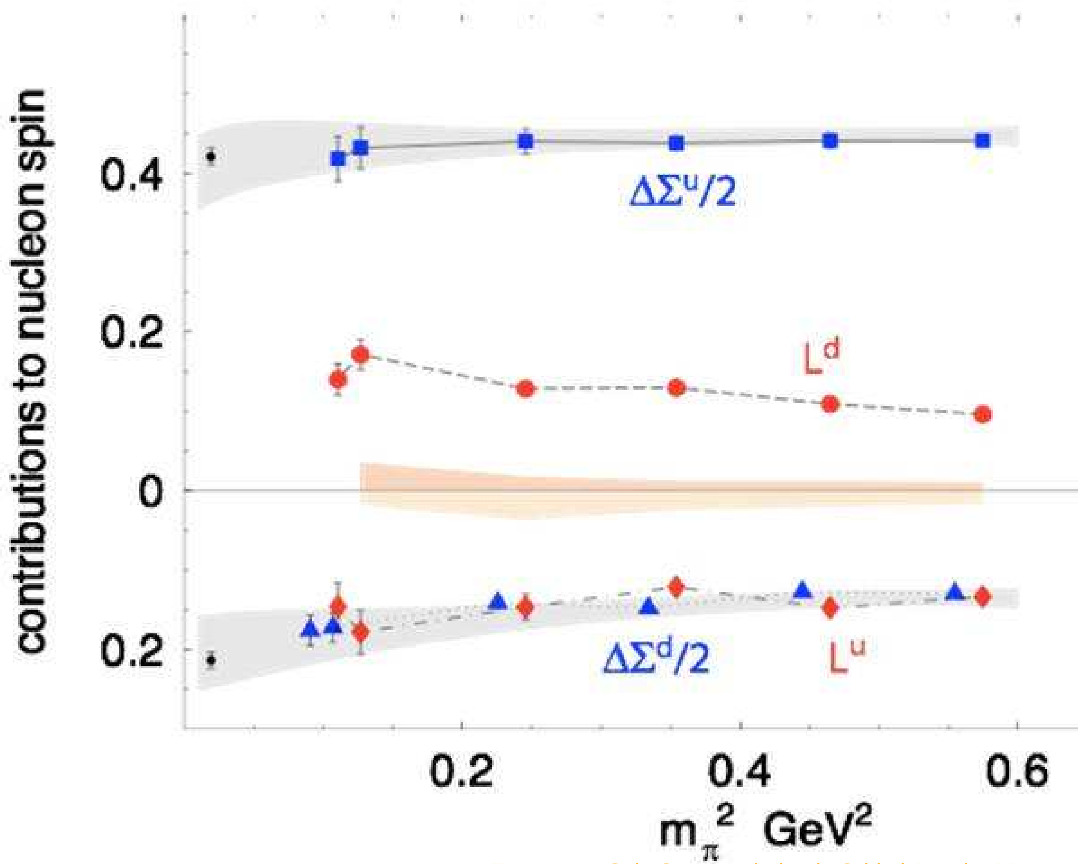
- lattice QCD $\Rightarrow$ moments of GPDs (LHPC; QCDSF)

$\hookrightarrow$ insert in Ji-relation

$$\langle J_q^i \rangle = S^i \int dx \left[H_q(x,0) + E_q(x,0) \right] x.$$

$\hookrightarrow$ $L_q^z = J_q^z - \frac{1}{2}\Delta q$

- L_u, L_d both large!
- present calcs. show $L_u + L_d \approx 0$, but
  - disconnected diagrams ..?
  - m_π^2 extrapolation
  - parton interpret. of L_q...



Angular Momentum in QCD (Jaffe & Manohar)

- define OAM on a light-like hypesurface rather than a space-like hypersurface

$$\tilde{J}^3 = \int d^2x_\perp \int dx^- M^{12+}$$

where $x^- = \frac{1}{\sqrt{2}}\left(x^0 - x^-\right)$ and $M^{12+} = \frac{1}{\sqrt{2}}\left(M^{120} + M^{123}\right)$

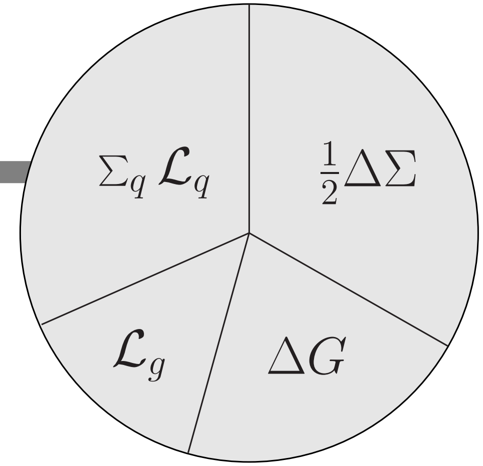
- Since $\partial_\mu M^{12\mu} = 0$

$$\int d^2\mathbf{x}_\perp \int dx^- M^{12+} = \int d^2\mathbf{x}_\perp \int dx^3 M^{120}$$

(compare electrodynamics: $\vec{\nabla} \cdot \vec{B} = 0 \quad \Rightarrow$ flux in = flux out)

- use eqs. of motion to get rid of 'time' (∂_+ derivatives) & integrate by parts whenever a total derivative appears in the T^{i+} part of M^{12+}

Jaffe/Manohar decomposition



- in light-cone framework & light-cone gauge $A^+ = 0$ one finds for $J^z = \int dx^- d^2\mathbf{r}_\perp M^{+xy}$

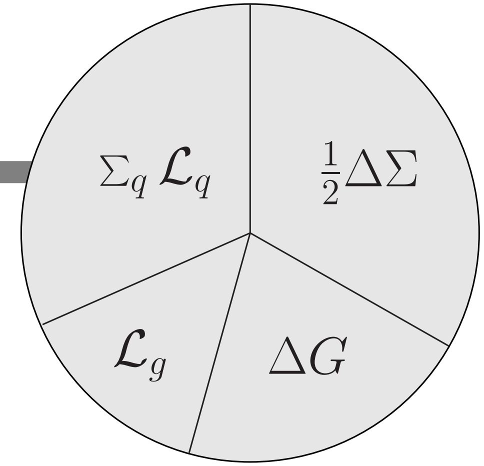
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

where ($\gamma^+ = \gamma^0 + \gamma^z$)

$$\begin{aligned}
\mathcal{L}_q &= \int d^3r \, \langle P,S| \, \bar{q}(\vec{r})\gamma^+ \left(\vec{r} \times i\vec{\partial}\right)^z q(\vec{r}) \, |P,S\rangle \\
\Delta G &= \varepsilon^{+-ij} \int d^3r \, \langle P,S| \, \mathrm{Tr} F^{+i} A^j \, |P,S\rangle \\
\mathcal{L}_g &= 2 \int d^3r \, \langle P,S| \, \mathrm{Tr} F^{+j} \left(\vec{x} \times i\vec{\partial}\right)^z A^j \, |P,S\rangle
\end{aligned}$$

Jaffe/Manohar decomposition

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$$



- $\Delta\Sigma = \sum_q \Delta q$ from polarized DIS (or lattice)
- ΔG from $\vec{p}\,\overleftarrow{p}$ or polarized DIS (evolution)

$\hookrightarrow$ ΔG gauge invariant, but local operator only in light-cone gauge

- $\int dx x^n \Delta G(x)$ for $n \geq 1$ *can* be described by manifestly gauge inv. local op. ($\longrightarrow$ lattice)
- $\mathcal{L}_q$, $\mathcal{L}_g$ independently defined, but
  - no exp. identified to access them
  - not accessible on lattice, since nonlocal except when $A^+ = 0$
- parton net OAM $\mathcal{L} = \mathcal{L}_g + \sum_q \mathcal{L}_q$ by subtr. $\mathcal{L} = \frac{1}{2} - \frac{1}{2}\Delta\Sigma - \Delta G$
- in general, $\mathcal{L}_q \neq L_q$ $\qquad$ $\mathcal{L}_g + \Delta G \neq J_g$
- makes no sense to 'mix' Ji and JM decompositions, e.g. $J_g - \Delta G$ has no fundamental connection to OAM

$L_q \neq \mathcal{L}_q$

- L_q matrix element of

$$q^\dagger \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q = \bar{q}\gamma^0 \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$$

- $\mathcal{L}_q^z$ matrix element of $(\gamma^+ = \gamma^0 + \gamma^z)$

$$\bar{q}\gamma^+ \left[\vec{r} \times i\vec{\partial}\right]^z q\Big|_{A^+=0}$$

- For nucleon at rest, matrix element of L_q same as that of $\bar{q}\gamma^+ \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$

$\hookrightarrow$ even in light-cone gauge, L_q^z and $\mathcal{L}_q^z$ still differ by matrix element of $q^\dagger \left(\vec{r} \times g\vec{A}\right)^z q\Big|_{A^+=0} = q^\dagger \left(xgA^y - ygA^x\right) q\Big|_{A^+=0}$

Summary part 1:

- Ji: $J^z = \frac{1}{2}\Delta\Sigma + \sum_q L_q + J_g$
- Jaffe: $J^z = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$
- ΔG can be defined without reference to gauge (and hence gauge invariantly) as the quantity that enters the evolution equations and/or $\vec{p}\,\overleftarrow{p}$

$\hookrightarrow$ represented by simple (i.e. local) operator only in LC gauge and corresponds to the operator that one would naturally identify with 'spin' only in that gauge

- in general $L_q \neq \mathcal{L}_q$ or $J_g \neq \Delta G + \mathcal{L}_g$, but
- how significant is the difference between L_q and $\mathcal{L}_q$, etc. ?

OAM in scalar diquark model

[M.B. + Hikmat Budhathoki Chhetri (BC), PRD **79**, 071501 (2009)]

- toy model for nucleon where nucleon (mass M) splits into quark (mass m) and scalar 'diquark' (mass λ)

$\hookrightarrow$ light-cone wave function for quark-diquark Fock component

$$\psi^{\uparrow}_{+\frac{1}{2}}(x, \mathbf{k}_\perp) = \left(M + \frac{m}{x}\right)\phi \qquad \psi^{\uparrow}_{-\frac{1}{2}} = -\frac{k^1 + ik^2}{x}\phi$$

with $\phi = \frac{c/\sqrt{1-x}}{M^2 - \frac{\mathbf{k}_\perp^2 + m^2}{x} - \frac{\mathbf{k}_\perp^2 + \lambda^2}{1-x}}$.

- quark OAM according to JM: $\mathcal{L}_q = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}(1-x)\left|\psi^{\uparrow}_{-\frac{1}{2}}\right|^2$
- quark OAM according to Ji: $L_q = \frac{1}{2}\int_0^1 dx\, x\left[q(x) + E(x,0,0)\right] - \frac{1}{2}\Delta q$

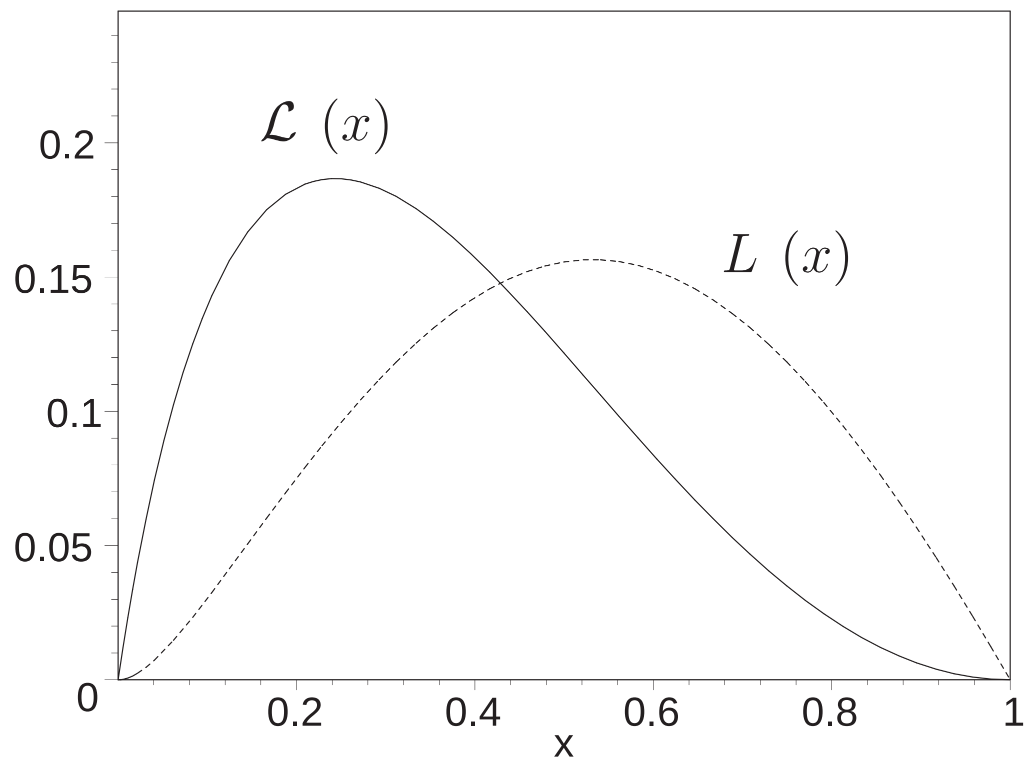
$\rightsquigarrow$ (using Lorentz inv. regularization, such as Pauli Villars subtraction) both give identical result, i.e. $L_q = \mathcal{L}_q$

- not surprising since scalar diquark model is <u>not</u> a gauge theory

OAM in scalar diquark model

- But, even though $L_q = \mathcal{L}_q$ in this non-gauge theory

$$\mathcal{L}_q(x) \equiv \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}(1-x)\left|\psi^{\uparrow}_{-\frac{1}{2}}\right|^2 \neq \frac{1}{2}\left\{x\left[q(x)+E(x,0,0)\right]-\Delta q(x)\right\} \equiv L_q(x)$$



$\hookrightarrow$ ‘unintegrated Ji-relation’ does <u>not</u> yield x-distribution of OAM

OAM in QED

- light-cone wave function in $e\gamma$ Fock component

$$\Psi^{\uparrow}_{+\frac{1}{2}+1}(x,\mathbf{k}_\perp) = \sqrt{2}\frac{k^1 - ik^2}{x(1-x)}\phi \qquad \Psi^{\uparrow}_{+\frac{1}{2}-1}(x,\mathbf{k}_\perp) = -\sqrt{2}\frac{k^1 + ik^2}{1-x}$$

$$\Psi^{\uparrow}_{-\frac{1}{2}+1}(x,\mathbf{k}_\perp) = \sqrt{2}\left(\frac{m}{x} - m\right)\phi \qquad \Psi^{\uparrow}_{-\frac{1}{2}+1}(x,\mathbf{k}_\perp) = 0$$

- OAM of e^- according to Jaffe/Manohar

$$\mathcal{L}_e = \int_0^1 dx \int d^2\mathbf{k}_\perp \left[(1-x)\left|\Psi^{\uparrow}_{+\frac{1}{2}-1}(x,\mathbf{k}_\perp)\right|^2 - \left|\Psi^{\uparrow}_{+\frac{1}{2}+1}(x,\mathbf{k}_\perp)\right|^2\right]$$

- e^- OAM according to Ji $L_e = \frac{1}{2}\int_0^1 dx\, x\left[q(x) + E(x,0,0)\right] - \frac{1}{2}\Delta q$

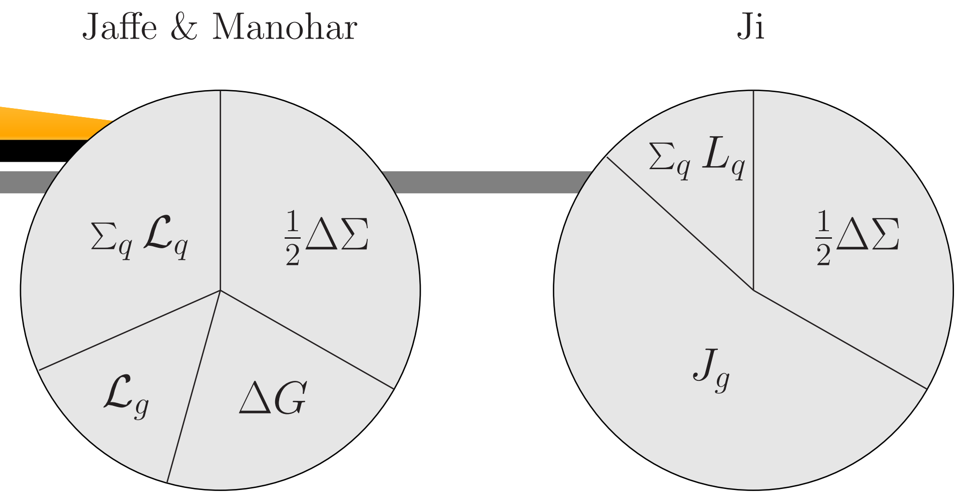
$\leadsto$ $\mathcal{L}_e = L_e + \frac{\alpha}{4\pi} \neq L_e$

- Likewise, computing J_γ from photon GPD, and $\Delta\gamma$ and $\mathcal{L}_\gamma$ from light-cone wave functions and <u>defining</u> $\hat{L}_\gamma \equiv J_\gamma - \Delta\gamma$ yields $\hat{L}_\gamma = \mathcal{L}_\gamma + \frac{\alpha}{4\pi} \neq \mathcal{L}_\gamma$
- $\frac{\alpha}{4\pi}$ appears to be small, but here $\mathcal{L}_e$, L_e are all of $\mathcal{O}(\frac{\alpha}{\pi})$

OAM in QCD

- $\hookrightarrow$ 1-loop QCD: $\mathcal{L}_q - L_q = \frac{\alpha_s}{3\pi}$
- recall (lattice QCD): $L_u \approx -.15$; $L_d \approx +.15$
- QCD evolution yields negative correction to L_u and positive correction to L_d
- $\hookrightarrow$ evolution suggested (A.W.Thomas) to explain apparent discrepancy between quark models (low Q^2) and lattice results ($Q^2 \sim 4GeV^2$)
- above result suggests that $\mathcal{L}_u > L_u$ and $\mathcal{L}_d > L_d$
- additional contribution (with same sign) from vector potential due to spectators (MB, to be published)
- $\hookrightarrow$ possible that lattice result consistent with $\mathcal{L}_u > \mathcal{L}_d$

Summary



- inclusive $\overrightarrow{e}\,\overleftarrow{p}/\overrightarrow{p}\,\overleftarrow{p}$ provide access to
  - quark spin $\frac{1}{2}\Delta q$
  - gluon spin ΔG
  - parton grand total OAM $\mathcal{L} \equiv \mathcal{L}_g + \sum_q \mathcal{L}_q = \frac{1}{2} - \Delta G - \sum_q \Delta q$
- DVCS & polarized DIS and/or lattice provide access to
  - quark spin $\frac{1}{2}\Delta q$
  - J_q & $L_q = J_q - \frac{1}{2}\Delta q$
  - $J_g = \frac{1}{2} - \sum_q J_q$
- $J_g - \Delta G$ does not yield gluon OAM $\mathcal{L}_g$
- $L_q - \mathcal{L}_q = \mathcal{O}(0.1 * \alpha_s)$ for O (α_s) dressed quark

Announcement:

- workshop on Orbital Angular Momentum of Partons in Hadrons
- ECT* 9-13 November 2009
- organizers: M.B. & Gunar Schnell
- confirmed participants: M.Anselmino, H.Avakian, A.Bacchetta, L.Bland, D.Boer, S.J.Brodsky, M.Diehl, D.Fields, L.Gamberg, G.Goldstein, M.Grosse-Perdekamp, P.Hägler, X.Ji, R.Kaiser, E.Leader, N.Makins, A.Miller, D.Müller, P.Mulders, A.Schäfer, G.Schierholz, O.Teryaev, W.Vogelsang, F.Yuan